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COMMENT

Comment on 'Self-dressing and radiation reaction in classical electrodynamics'*

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Abstract

Using the canonical formalism, Compagno and Persico (2002 *J. Phys. A: Math. Gen.* **35** 3629–45) have calculated the 'radiation-reaction' force on a uniform spherical charge moving rigidly, slowly and slightly from its position at the time when the transverse electric field is assumed to vanish. This force is shown to result in the same time-averaged self-force as that which has been obtained by different means for the test charge of a Bohr–Rosenfeld field-measurement procedure and which Compagno and Persico claimed to be incorrect.

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In a recent paper [1], Compagno and Persico (CP) have calculated, by solving the coupled charge–field Hamilton equations of motion, the 'radiation-reaction' force on a spherically symmetric charge that moves rigidly, slowly and only a little from its position at the time when the transverse electric field is assumed to vanish. For a charge q that is uniformly distributed within a sphere of radius a, they obtain a 'radiation-reaction' force

$$F_{\mathrm{RR}}(t) = -\frac{2q^2}{a^3}\int_0^t \mathrm{d}t'\,\dot{Q}(t')\left[1 - \frac{3(t-t')}{2a} + \frac{(t-t')^3}{4a^3}\right]\Theta[2a-(t-t')] \qquad (1)$$

where $\dot{Q}(t)$ is the time derivative of a one-dimensional trajectory of the charge and $\Theta(x)$ is the Heaviside step function; here and henceforth, we use units such that the speed of light $c = 1$ and put $t = 0$ for the time at which the transverse electric field vanishes.

CP remark that result (1) is relevant to the issues raised in recent papers [2–5] in connection with the Bohr–Rosenfeld (BR) analysis of the measurability of the electromagnetic field [6], as it should apply to a BR measurement procedure with only minor modifications. In this comment, we show that the force (1), which we prefer to call the electromagnetic self-force, results in the same time-averaged self-force as that which has been obtained by different means in [3, 5] for the test charge of a BR measurement procedure and which CP have rejected in [4] as incorrectly calculated.

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A condition on the one-dimensional trajectory $Q(t)$ of the test charge in a BR measurement procedure occupying a time interval $(0, T)$ is that $Q(t) = 0$ for $t \leqslant 0$. While this means that the transverse electric field of the test charge vanishes at $t = 0$, the initial condition $\dot{Q}(t)|_{t=0} = 0$ that is implied would result according to (1) in $Q(t) = 0$ also for $t > 0$ if there were no other force acting on the test charge in addition to the self-force. It is presumably for this reason that when CP touch on the applicability of (1) to the test charge of the BR measurement procedure, they invoke the 'neutralization' of the test charge at $t = 0$ by the stationary neutralizing charge employed in the procedure. Besides the question whether such a neutralization alone indeed guarantees a vanishing transverse electric field at $t = 0$, the fact remains that then there is at least one other force acting on the test charge, namely the electrostatic force of attraction to the neutralizing charge. It thus appears inescapable that any meaningful use of formula (1) in an analysis of the BR field-measurement procedure requires the presence of external forces. We shall first write (1) in a different form before returning to this point.

Using integration by parts, we write (1) as

$$F_{\mathrm{RR}}(t) = Q(t')u(t-t')\big|_{t'=0}^{t} - \int_0^t \mathrm{d}t'\, Q(t') \frac{\mathrm{d}u(t-t')}{\mathrm{d}t'} \tag{2}$$

where

$$u(t-t') = -\frac{2q^2}{a^3}\left[1 - \frac{3(t-t')}{2a} + \frac{(t-t')^3}{4a^3}\right]\Theta[2a-(t-t')]. \tag{3}$$

Now, $u(t-t')|_{t'=t} = -2q^2/a^3$ and

$$\frac{\mathrm{d}u(t-t')}{\mathrm{d}t'} = -\frac{3q^2}{2a^4}\left[2 - \frac{(t-t')^2}{a^2}\right]\Theta[2a-(t-t')]. \tag{4}$$

Using this and an initial condition $Q(t')|_{t'=0} = 0$, we obtain the self-force (2) for $t < T$ as

$$F_{\mathrm{RR}}(t) = -\frac{2q^2}{a^3}Q(t) + \frac{3q^2}{2a^4}\int_0^T \mathrm{d}t'\, Q(t')\left[2 - \frac{(t-t')^2}{a^2}\right]\Theta(t-t')\Theta[2a-(t-t')] \tag{5}$$

where the factor $\Theta(t-t')$ is introduced in the integrand in order to fix the integration range as $(0, T)$. Apart from differences in notation, this expression is the same as that for the self-force $F_x(t_2)$ given in [5] by equations (26) and (28), with a factor of 2 instead of 3 in the delta-function term of equation (28) so that the electrostatic force due to a BR neutralizing charge is not included.

The self-force $F_x(t_2)$ was obtained in [5] by using the electromagnetic self-field of a uniform spherical charge whose trajectory was *prescribed* to be a given trajectory $Q(t)$ satisfying the conditions imposed on it by the BR field-measurement procedure (but not necessarily the stipulation that it is to have a step-like character with respect to the measurement period $(0, T)$). This fact invalidates the caution of CP that the force which they obtained 'is unambiguously the radiation-reaction force only in the absence of other forces'. A prescribed trajectory can eventuate to a given degree of accuracy only when there is a suitable external force acting on the charge in addition to the self-force. In the BR field-measurement procedure, the test charge is acted on by an external force that is the resultant of forces originating from several sources: the momentum-measurement system, the neutralizing charge, the spring that compensates the time-averaged effects of the self-force and the neutralizing charge, and, of course, the measured external field itself; the approximate attainment of the prescribed step-like trajectory is facilitated there by the fact that the test charge is allowed to have an arbitrarily great mass.

The self-force averaged over a time T,

$$\bar{F}_{\mathrm{RR}} = \frac{1}{T}\int_0^T \mathrm{d}t\, F_{\mathrm{RR}}(t) \tag{6}$$

can now be written using (5) as

$$\bar{F}_{\mathrm{RR}} = -\frac{2q^2}{a^3T}\int_0^T \mathrm{d}t\, Q(t) + \frac{q^2}{T}\int_0^T \mathrm{d}t'\, Q(t')\int_0^T \mathrm{d}t\, v(t-t') \tag{7}$$

where

$$v(t-t') = \frac{3}{2a^4}\left[2 - \frac{(t-t')^2}{a^2}\right]\Theta(t-t')\Theta[2a-(t-t')]. \tag{8}$$

The integration with respect to t in the second term of (7) is straightforward, yielding

$$\int_0^T \mathrm{d}t\, v(t-t') = \frac{2}{a^3} + f(t') \tag{9}$$

where

$$f(t') = -\frac{1}{2a^3}(2-\chi)(2-2\chi-\chi^2)\Theta(2-\chi) \qquad \chi = \frac{T-t'}{a}. \tag{10}$$

Using (9) in (7), the time-averaged self-force is obtained finally as

$$\bar{F}_{\mathrm{RR}} = \frac{q^2}{T}\int_0^T \mathrm{d}t'\, Q(t')f(t'). \tag{11}$$

Expressions (10) and (11) are, apart from differences in notation and the absence in (10) of the electrostatic term $-1/a^3$ due to the neutralizing charge, the same as those for the time-averaged self-force on the test charge of a BR-like measurement procedure given in equations (3) and (9) of [3] and equations (11) and (22) of [5]; these expressions have been claimed by CP to be incorrect [4].

It has been shown in [5] that the rejection [4] of CP of the criticism [3] of their re-analysis [2] of the BR field-measurement procedure is based on erroneous calculations. It has to be concluded that this rejection is invalidated also by the results of the latest work of CP themselves.

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